can be written as

$$U_1=\begin{pmatrix} (1+\cos\beta)/2 & -(\sin\beta)/\sqrt{2} & (1-\cos\beta)/2 \\ (\sin\beta)/\sqrt{2} & \cos\beta & -(\sin\beta)/\sqrt{2} \\ (1-\cos\beta)/2 & (\sin\beta)/\sqrt{2} & (1+\cos\beta)/2 \end{pmatrix}.$$

Another Lorentz transformation in the new z direction to the rest frame of [1] does not change the helicity state.

The justification of the unphysical transformation can be seen in two ways. From dispersion theory, $\cos\beta$ is the same as the cosine of the angle in the t channel. If, on the other hand, we construct the field function of [3] and impose the Lorentz condition even in the case that [3] is virtual, to eliminate the spin-zero component, we obtain the same answer for the transformation between the field functions in two vertices.

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S-Wave Hyperon-Nucleon Interactions and SU_3 Symmetry

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The generalized Pauli principle is combined with the assumption of SU_3 symmetry to yield relations between hyperon-nucleon and nucleon-nucleon scattering amplitudes for the $^1S_0$ and $^3S_1$ states.

Although many applications of the "eightfold way" version of unitary symmetry[1] have been made to two-body meson baryon reactions, relatively little attention has been paid to baryon-baryon systems.[2] In this note we combine the assumption of SU_3 symmetry with the generalized Pauli principle to deduce relations between hyperon-nucleon and nucleon-nucleon amplitudes.[3] Particular attention is given to those reactions which are most readily accessible to experiments, namely,

$$p+p\rightarrow p+p \quad T(pp) \tag{1a}$$

$$n+p\rightarrow n+p \quad T(nn) \tag{1b}$$

$$\Sigma^++p\rightarrow \Sigma^++p \quad T(\Sigma^+\Sigma^+) \tag{1c}$$

$$\Sigma^-+p\rightarrow \Sigma^-+p \quad T(\Sigma^-\Sigma^-) \tag{1d}$$

$$\Sigma^-+p\rightarrow \Sigma^0+n \quad T(\Sigma^-\Sigma^0) \tag{1e}$$

$$\Sigma^-+p\rightarrow \Lambda+n \quad T(\Sigma^-\Lambda) \tag{1f}$$

$$\Lambda+p\rightarrow \Lambda+p \quad T(\Lambda\Lambda) \tag{1g}$$

$$\Lambda+p\rightarrow \Sigma^++n \quad T(\Lambda\Sigma^+) \tag{1h}$$

$$\Lambda+p\rightarrow \Sigma^0+p \quad T(\Lambda\Sigma^0) \tag{1i}$$

In general, the wave function of two particles, each of which belongs to an octet representation of SU_3, will be a linear combination of irreducible wave functions belonging to the representations [27], [8_s], [8_a], [10], [$\overline{10}$], and [1]. (Note that the symbol [$\overline{10}$] denotes a continuous bar over the "one" and "zero.") The generalized Pauli principle applied to states containing two baryons which belong to the $J^P=1/2^+$ baryon octet, allows a reduction in the number of independent, reduced SU_3 matrix elements needed to describe the reactions of Eq. (1).

The total wave function for two baryons must be antisymmetric under the interchange of all of the coordinates of the two particles. In the two nucleon problem it is customary to split the total wave function into several parts, one describing the isospin and the other the spin-space part. However, if SU_3 invariance is assumed, then the dichotomy is into an SU_3 part and a spin-space part. Correspondingly, when two baryons are in an antisymmetric spin-space state ($^1S_0$, $^3P_{0,1,2}$, $\cdots$ in the notation $^{2S+1}L_J$), their SU_3 wave function $[(B_1B_2+B_2B_1)/\sqrt{2}]$ must be symmetric. B_1 and B_2 represent the SU_3 functions for baryons 1 and 2, respectively. Similarly, for symmetric spin-space states ($^3S_1$, $^1P_1$, $\cdots$) the SU_3 wave function $[(B_1B_2-B_2B_1)/\sqrt{2}]$ is antisymmetric. In general, the SU_3 symmetric states belong to the representations [27], [8_s], and [1], although in the reactions of Eq. (1) the singlet [1] state does not occur. The antisymmetric SU_3 states are contained in [8_a], [10], and [$\overline{10}$].

Assuming strict SU_3 invariance of the strong inter-

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[2] R. J. Oakes, Phys. Rev. **131**, 2239 (1963); I. Gerstein (preprint).
[3] For an example of the combination of generalized Bose symmetry with SU_3 invariance as applied to mesons, see C. A. Levinson, H. J. Lipkin and S. Meshkov, Phys. Letters **7**, 81 (1963).

TABLE I. Matrix elements for the reactions of Eq. (1) expressed in terms of SU_3 invariant reduced matrix elements for $^1S_0$ and $^3S_1$ incident or final states. The symbols $|B_1B_2\rangle$ are abbreviations for $|(B_1B_2+B_2B_1)/\sqrt{2}\rangle$ and $|B_1B_2-B_2B_1)/\sqrt{2}\rangle$.

$^1S_0$		$^3S_1$	
a. $\langle pp\|T^0\|pp\rangle$	$2T_{27}$		
b. $\langle np\|T^0\|np\rangle$	T_{27}	$\langle np\|T^1\|np\rangle$	$T_{\overline{10}}$
c. $\langle\Sigma^+p\|T^0\|\Sigma^+p\rangle$	T_{27}	$\langle\Sigma^+p\|T^1\|\Sigma^+p\rangle$	T_{10}
d. $\langle\Sigma^-p\|T^0\|\Sigma^-p\rangle$	$(1/5)(2T_{27}+3T_{8_s})$	$\langle\Sigma^-p\|T^1\|\Sigma^-p\rangle$	$(1/3)(T_{\overline{10}}+T_{10}-T_{8_a})$
e. $\langle\Sigma^0n\|T^0\|\Sigma^-p\rangle$	$(3/5\sqrt{2})(T_{27}-T_{8_s})$	$\langle\Sigma^0n\|T^1\|\Sigma^-p\rangle$	$(1/3\sqrt{2})(T_{\overline{10}}-2T_{10}+T_{8_a})$
f. $\langle\Lambda n\|T^0\|\Sigma^-p\rangle$	$(\sqrt{3}/5\sqrt{2})(T_{27}-T_{8_s})$	$\langle\Lambda n\|T^1\|\Sigma^-p\rangle$	$(1/\sqrt{6})(-T_{\overline{10}}+T_{8_a})$
g. $\langle\Lambda p\|T^0\|\Lambda p\rangle$	$(1/10)(9T_{27}+T_{8_s})$	$\langle\Lambda p\|T^1\|\Lambda p\rangle$	$(1/2)(T_{\overline{10}}+T_{8_a})$
h. $\langle\Sigma^+n\|T^0\|\Lambda p\rangle$	$(\sqrt{3}/5\sqrt{2})(T_{27}-T_{8_s})$	$\langle\Sigma^+n\|T^1\|\Lambda p\rangle$	$(1/\sqrt{6})(T_{\overline{10}}-T_{8_a})$
i. $\langle\Sigma^0p\|T^0\|\Lambda p\rangle$	$(\sqrt{3}/10)(T_{27}-T_{8_s})$	$\langle\Sigma^0p\|T^1\|\Lambda p\rangle$	$(1/2\sqrt{3})(-T_{\overline{10}}+T_{8_a})$

actions, the reaction amplitudes of Eq. (1) can be expressed as linear combinations of two independent reduced amplitudes, T_{27}, T_{8_s} in the $^1S_0$ state, and three reduced amplitudes T_{8_a}, T_{10}, $T_{\overline{10}}$, in the $^3S_1$ state. This is a great simplification compared to that which prevails for reactions like meson+baryon $\rightarrow$ meson+baryon.[4,5] For arbitrary angular momenta, the five reduced amplitudes listed above must be augmented by another independent amplitude, $T_{8_{sa}}$, that couples a two baryon $[8_a]$ state to an $[8_s]$ state, or vice versa. This mixing amplitude can occur only for states in which $J=L$ (J and L are the total and orbital angular momentum quantum numbers, respectively). In such states, J and parity can be conserved in a transition of the type $[8_s]\leftrightarrow[8_a]$, for example: $[8_a]\,^1P_1\leftrightarrow[8_s]\,^3P_1$. Transitions of this type are forbidden for $n-p$ scattering by charge independence when the generalized Pauli principle is invoked. However, these transitions are *not* forbidden by SU_3 invariance for hyperon-nucleon reactions, without an additional postulate such as R invariance, which apparently does not hold for strong reactions.[4] Because of the effect of a possible nonvanishing $T_{8_{sa}}$ amplitude we restrict our discussion to S waves.

The Clebsch-Gordan coefficients that couple symmetric and antisymmetric two-baryon states to SU_3 invariant states have been listed in the literature.[6] Table I lists the amplitudes of reactions (1) in terms of the SU_3 reduced matrix elements, for both $^1S_0$ and $^3S_1$ states. It is convenient to adopt the following notation: $T^0(B_1B_2)$ and $T^1(B_1B_2)$ are the transition matrix elements for reactions of the form $B_1+p\rightarrow B_2+p$ or $B_1+p\rightarrow B_2+n$ in the $^1S_0$ or $^3S_1$ initial state, respectively.

Table I yields the following equalities for the $^1S_0$ state:

$$(1/4)|T^0(pp)|^2=|T^0(nn)|^2=|T^0(\Sigma^+\Sigma^+)|^2 \quad (2)$$

$$|T^0(\Sigma^-\Sigma^0)|^2=3|T^0(\Sigma^-\Lambda)|^2=3|T^0(\Lambda\Sigma^+)|^2 =6|T^0(\Lambda\Sigma^0)|^2 \quad (3)$$

and

$$|T^0(nn)|^2+(1/5)|T^0(\Sigma^-\Sigma^-)|^2 =(6/5)|T^0(\Lambda\Lambda)|^2+(1/3)|T^0(\Sigma^-\Sigma^0)|^2. \quad (4)$$

For the $^3S_1$ state

$$|T^1(\Sigma^-\Lambda)|^2=|T^1(\Lambda\Sigma^+)|^2=2|T^1(\Lambda\Sigma^0)|^2 \quad (5)$$

and

$$3[|T^1(\Sigma^-\Sigma^-)|^2+|T^1(\Sigma^-\Sigma^0)|^2] =2|T^1(\Lambda\Lambda)|^2+|T^1(\Sigma^+\Sigma^+)|^2. \quad (6)$$

In addition, the relations in Table I imply many inequalities, such as

$$3|T^1(\Sigma^-\Lambda)|^2\geq|T^1(nn)|^2-|T^1(\Lambda\Lambda)|^2, \quad (7)$$

$$3|T^1(\Sigma^-\Lambda)|^2\geq2|T^1(nn)|^2+2|T^1(\Lambda\Lambda)|^2 -4|T^1(nn)||T^1(\Lambda\Lambda)|, \quad (8)$$

$$3|T^1(\Sigma^-\Lambda)|^2\leq2|T^1(nn)|^2+2|T^1(\Lambda\Lambda)|^2 +4|T^1(nn)||T^1(\Lambda\Lambda)|. \quad (9)$$

Equation (2) is valid for all antisymmetric spin-space states ($^1S_0$, $^3P_{0,1,2,\ldots}$) since the pp, nn, and $\Sigma^+\Sigma^+$ amplitudes are not coupled to either the $[8_s]$ or $[8_a]$ representation.

The difficult question remains as to what relation, if any, Eqs. (2)–(9) have with experimental S-wave cross sections, since SU_3 symmetry is broken. It clearly makes no sense to compare the cross sections of endothermic reactions, such as $\Lambda+p\rightarrow\Sigma^++n$, with the exothermic reactions like $\Sigma^-+p\rightarrow\Lambda+n$ at a kinetic energy where the former is not energetically allowed. In the S-wave dominant region, one might hope to compare the first seven reactions of Eq. (1) by choosing the same kinetic energy in the incident state (or what is almost but not quite equivalent, the same momentum in the incident state). Since these reactions are either elastic or exothermic, this criterion has the virtue that all of the

[4] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 1, 44 (1962).
[5] P. G. O. Freund, H. Ruegg, D. Speiser, and A. Morales, Nuovo Cimento 25, 307 (1962).
[6] J. J. deSwart, Rev. Mod. Phys. 35, 916 (1963); R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. 34, 1 (1962).

channels are simultaneously open. An analogous problem for a different class of *endothermic* reactions was analyzed successfully by Meshkov, Snow, and Yodh,[7] who compared different endothermic reactions at the same outgoing kinetic energy.

In the low-energy region, the rather large $\Sigma^0-\Lambda$ mass difference may cause large deviations from the pure SU_3 predictions, for reactions (1e) and (1f). For example, if tensor forces are important[8] for an incident $^3S_1(\Sigma^-p)$ state, the outgoing $^3D_1$ state of Σ^0n will be strongly suppressed by centrifugal barrier effects relative to the outgoing $^3D_1$ state of Λn.[8]

A particularly interesting comparison may be made between the cross sections for the processes $n+p\to n+p$ and $\Sigma^++p\to\Sigma^++p$. Their $^1S_0$ cross sections both depend only on T_{27} and should be the same. However, the $^3S_1$ cross section for the Σ^+p system depends on T_{10}, whereas the $^3S_1$ system for the $n+p$ system corresponds to the deuteron ($T_{\overline{10}}$). Since

$$\sigma_{\text{tot}}(\Sigma^+\Sigma^+)=(1/4)\sigma^0(\Sigma^+\Sigma^+)+(3/4)\sigma^1(\Sigma^+\Sigma^+) \quad (10)$$

and

$$\sigma^0(nn)=\sigma^0(\Sigma^+\Sigma^+)\,, \quad (11)$$

SU_3 invariance predicts that

$$\sigma_{\text{tot}}(\Sigma^+\Sigma^+)>(1/4)\sigma^0(nn)\,. \quad (12)$$

The amount by which $\sigma_{\text{tot}}(\Sigma^+\Sigma^+)$ is larger than $\sigma^0(nn)$ is a direct measure of T_{10}. A difficulty with this analysis arises if we consider the hyperon-nucleon potential as arising from meson exchange. The wide variation of the masses of the eight pseudoscalar mesons would imply substantial differences in the ranges of parts of the hyperon-nucleon potential compared to those of the nucleon-nucleon potential.[9] This might produce deviations from the SU_3 prediction given above.

Despite all of the difficulties cited above, comparison of the reactions (1) with Eqs. (2)–(9) should prove useful because it may provide important clues about the effect of SU_3 symmetry breaking on baryon-baryon dynamics. The S-wave cross sections for the reactions Eqs. (1a)–(1g) are all observable, since K^- mesons stopping in a hydrogen bubble chamber provide an excellent source of low-energy Σ^+, Σ^-, and Λ hyperons. The interactions of these hyperons with protons can be studied in the same pictures which record their production.[10]

Note added in proof. Preliminary experimental results of R. Burnstein *et al.*[10] yield $\sigma_{\text{tot}}(\Sigma^+,\Sigma^+)=200\pm100$ mb at a Σ^+ average laboratory momentum of 160 MeV/c. The assumption of SU_3 invariance combined with p-p scattering data predicts $\frac{1}{4}\sigma^0(\Sigma^+,\Sigma^+)=165$ mb at this momentum, indicating that $\sigma^1(\Sigma^+,\Sigma^+)$ is small.

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Interpretation of High-Energy Large-Angle Scattering*

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It is shown that if the analytically continued partial-wave amplitude is assumed to have l dependence

$$a_\pm(s,l)=\sum_{l=0}^{n} C_m{}^\pm(s)l^m(l+1)^m$$

for $l<l_0(s)$ and finite n, the scattering amplitude is bounded by $\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{\frac{1}{2}}\}$ at high energies. Here $a_+(s,l)[a_-(s,l)]$ is equal to $a_l(s)$ for even (odd) integer l. The most physical example of this dependence is that in which a central area of the scatterer becomes maximally absorptive.

THE large angle p–p elastic-scattering cross section[1] shows a strong dependence on both energy and momentum transfer. Orear[2] has pointed out that this strong dependence can be fitted by a single exponential in the transverse momentum. If this dependence holds to arbitrarily high energies, the scattering amplitude for a fixed angle must decrease for increasing energy as $\exp(-\text{const}\ s^{1/2})$, where s is the square of the center-of-mass energy. At any rate it appears that the scattering amplitude for finite fixed angle is a rapidly decreasing function of s.

The purpose of this note is to show that this rapid decrease of the scattering amplitude at finite angles

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